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DEVELOPMENT OF A RECTILINEAR AXISYMMETRIC VISCOPLASTIC FLOW AND ELASTIC AFTEREFFECT AFTER ITS STOP

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An analytical solution is given for the boundary-value problem of the theory of large elastoviscoplastic strains for rectilinear motion of a material in the gap between two rigid coaxial cylindrical surfaces. The cases of initially uniformly accelerated motion with the subsequent uniformly decelerated stop of one of the surfaces when the other surface remains motionless are considered. The laws of motion of the elastoplastic boundaries and distributions of displacements and stresses, including residual ones, are given.

Key words: *elasticity, viscoplasticity, large strains, residual stress.*

Rectilinear motions of viscoplastic media were considered in [1–3]. Due to the nonlinearity following from the model of system of equations, a few analytical solutions have been obtained even ignoring the elastic properties of the medium within the Shvedov–Bingham model. Generally, elastic strains are considered negligibly small compared to plastic strains. However, during intense distortion of bodies being deformed (for example, during pressure working of metals), reversible strains can lead to significant changes in the shape and volume of workpieces being processed after removal of tooling (during unloading) and to an inadmissible increase in the residual stress arising in fabricated parts. These effects can be studied only using the model of large elastoviscoplastic strains because the irreversible strains in these processes cannot be considered small.

This paper gives a solution of the boundary-value problem of the theory of large elastoviscoplastic strains for rectilinear motion of a material located between rigid coaxial cylindrical surfaces, one of which moves. The problem is solved in a quasistatic formulation. A method for the solution of such problems is proposed which can be used in the simplest case (where the reversible strains are small so that their cubes can be neglected in the calculation of stresses) to determine the position of the elastoplastic boundaries at each time and obtain analytical relations for displacements in both the region of reversible deformation and the region of viscoplastic flow, without complex calculations. We note that this method takes into account the properties of the model of large elastoplastic strains [4, 5], which was extended to the case of accounting for the viscous properties of materials in plastic flow [6].

1. Basic Relations of the Model. We assume that reversible and irreversible strains are independent thermodynamic parameters of the state of a deformable medium which are described by the equations of variation (transfer) [7] defining the strains components not measured experimentally. As in [4, 5], assuming that coordinate system is rectangular and Cartesian, we write these equations in the Euler spatial variables x_i:

$$\frac{De_{ij}}{Dt} = \varepsilon_{ij} - \varepsilon_{ij}^p - \frac{1}{2}\Big((\varepsilon_{ik} - \varepsilon_{ik}^p + z_{ik})e_{kj} + e_{ik}(\varepsilon_{kj} - \varepsilon_{kj}^p - z_{kj})\Big),$$

$$\frac{Dp_{ij}}{Dt} = \varepsilon_{ij}^p - p_{ik}\varepsilon_{kj}^p - \varepsilon_{ik}^p p_{kj}, \tag{1.1}$$

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$$\frac{Dn_{ij}}{Dt} = \frac{dn_{ij}}{dt} - r_{ik}n_{kj} + n_{ik}r_{kj}, \qquad \varepsilon_{ij} = \frac{1}{2}\,(v_{i,j} + v_{j,i}),$$

$$r_{ij} = \frac{1}{2}\,(v_{i,j} - v_{j,i}) + z_{ij}(\varepsilon_{sk}, e_{sk}), \qquad v_i = \frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + u_{i,j}v_j, \qquad u_{i,j} = \frac{\partial u_i}{\partial x_j}.$$

Here p_{ij} and e_{ij} are the components of the irreversible and reversible strain tensor, respectively, and u_i and v_i are the components of the displacement and velocity vectors of points of the medium. In the equation of the irreversible strain tensor, the source ε^p_{ij} is identified with the rate of change of irreversible (viscoplastic) strains. The presence of rotations r_{ij} of the nonlinear part z_{ij} in the tensor (in [4, 5], the relation $z_{ij}(\varepsilon_{sk}, e_{sk})$ is written completely) is due to the requirement of invariance of the irreversible strain tensor during unloading ($\varepsilon^p_{ij} = 0$). In this case, the irreversible strain tensor components p_{ij} vary in the same manner as during rigid body displacement. To satisfy this requirement and the laws of thermodynamics, it is necessary to use the objective derivative D/Dt which, in relations (1.1), is written for an arbitrary tensor n_{ij}.

The components of the Almansi strain tensor d_{ij} are linked to the components p_{ij} and e_{ij} by the relation

$$d_{ij} = e_{ij} + p_{ij} - e_{ik}e_{kj}/2 - e_{ik}p_{kj} - p_{ik}e_{kj} + e_{ik}p_{ks}e_{sj}. \tag{1.2}$$

The material will be considered incompressible. If the free-energy distribution density is determined only by the reversible strain, the first law of thermodynamics leads to analogs of the Murnaghan formula [4, 5]

$$\sigma_{ij} = \begin{cases} -p\delta_{ij} + \dfrac{\partial W}{\partial d_{ik}}\,(\delta_{kj} - 2d_{kj}), & p_{ij} \equiv 0, \\[2ex] -p_1\delta_{ij} + \dfrac{\partial W}{\partial e_{ik}}\,(\delta_{kj} - e_{kj}), & p_{ij} \neq 0, \end{cases} \tag{1.3}$$

where

$$W = -2\mu J_1 - \mu J_2 + bJ_1^2 + (b-\mu)J_1J_2 - \chi J_1^3 + \ldots, \qquad J_k = \begin{cases} L_k, & p_{ij} \equiv 0, \\ I_k, & p_{ij} \neq 0, \end{cases}$$

$$L_1 = d_{kk}, \quad L_2 = d_{ik}d_{ki}, \quad I_1 = e_{kk} - e_{sk}e_{ks}/2, \quad I_2 = e_{st}e_{ts} - e_{sk}e_{kt}e_{ts} + e_{sk}e_{kt}e_{tn}e_{ns}/4,$$

σ_{ij} are the Euler–Cauchy stress tensor components, p and p_1 are the additional hydrostatic pressures, W is the elastic potential, and μ, b, and χ are material constants.

As the plastic potential we use the generalized Tresca yield condition [8, 9]

$$\max\,|\sigma_i - \sigma_j| = 2k + 2\eta \max\,|\varepsilon^p_k|, \tag{1.4}$$

where σ_i and ε^p_k are the principal values of the plastic strain stress and rate tensors, η is the viscosity, and k is the yield point.

The irreversible strain rates are related to the stresses by the associated plastic flow law.

2. Elastic Equilibrium. Let an incompressible elastoviscoplastic material having the deformation properties considered above be located between two rigid coaxial cylindrical surfaces: a motionless outer surface of radius R and an inner surface of radius $r = r_0$ which moves along the z axis. In cylindrical coordinates (r, φ, z), the solution of this boundary-value problem is sought in the class of functions $u = u_z(r,t)$ and $v = v_z(r,t)$. The boundary conditions of the problem are written as

$$u(R,t) = v(R,t) = 0 \quad \forall\, t, \qquad u(r_0) = u_0 \quad \text{at} \quad t = 0,$$
$$v(r_0,t) = \alpha t \quad \text{at} \quad t \geqslant 0. \tag{2.1}$$

Thus, plastic flow near the inner rigid wall is assumed to begin at the time $t = 0$; before this time, the material is deformed reversibly. Let us calculate the parameters of this elastic equilibrium state, which is the initial condition for the subsequent process of irreversible deformation.

In the case considered, the nonzero components of the Almansi tensor are

$$d_{rr} = -\frac{1}{2}\,(u')^2, \qquad d_{rz} = \frac{1}{2}\,u', \qquad u' = \frac{\partial u}{\partial r}. \tag{2.2}$$

Using relations (1.1) and (2.2), for the stress components up to the second order of smallness in u', we obtain

$$\sigma_{rr} = \sigma_{\varphi\varphi} = -(p + 2\mu) - (b + \mu)(u')^2/2 = -s,$$

$$\sigma_{zz} = -s + \mu(u')^2, \qquad \sigma_{rz} = \mu u'.$$

According to equilibrium conditions

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{r} = 0, \qquad \frac{\partial \sigma_{rz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} = 0,$$

s is a function of only z; therefore, $s = az + a_0$. For the stresses σ_{rr}, $\sigma_{\varphi\varphi}$, and σ_{zz} to be finite as $z \to \infty$, it is necessary to set $a = 0$. Then, the solution of the elastic problem has the form

$$\sigma_{rz} = c/r, \qquad \sigma_{rr} = \sigma_{\varphi\varphi} = -a_0, \qquad \sigma_{zz} = -a_0 + c^2/(\mu r^2),$$
$$u = F(c, r), \qquad F(c, r) = (c/\mu)\ln(r/R). \tag{2.3}$$

To find the displacement component, we use the first boundary condition (2.1). The constant c is determined using the condition of arrival of the stress state at the loading surface (1.4). In the case considered, this condition is first satisfied on the surface $r = r_0$ in the form

$$\sigma_{rz}\Big|_{r=r_0} = -k.$$

From this, we obtain $c = -kr_0$. The constant a_0 influences only the distribution of the normal stress components (specified compression of the layer) and, hence, can be considered to the a specified quantity which can both remain constant and vary with time during the plastic flow process. According to relations (2.1), to initiate plastic flow on the inner surface, it should be moved a distance u_0 equal to $u_0 = (k/\mu) r_0 \ln(R/r_0)$.

According to relation (1.2), the reversible strain components are calculated from the obtained displacement field:

$$e_{rz} = d_{rz} = \frac{1}{2}\,u' = -\frac{kr_0}{2\mu r}, \qquad e_{rr} = -\frac{3}{2}\,e_{rz}^2, \qquad e_{zz} = \frac{1}{2}\,e_{rz}^2. \tag{2.4}$$

3. Viscoplastic Flow. Beginning at the time $t_0 = 0$, the inner rigid surface moves with velocity $v = \alpha t$. In this case, the developing region of viscoplastic flow is bounded by the surfaces $r = r_0$ and $r = r_1(t)$ $[r_0 \leqslant r \leqslant r_1(t)]$. In the region $r_1(t) \leqslant r \leqslant R$, the material is still deformed reversibly, i.e., the surface $r = r_1(t)$ is the moving boundary of the region of developing viscoplastic flow. We calculate the parameters of the stress–strain state corresponding to the velocity $v^* = \alpha t_1$ $(t_1 \geqslant 0)$ at $r = r_0$.

Integrating the equilibrium equations (quasistatic approximation) as above, in the region of reversible deformation $r_1(t) \leqslant r \leqslant R$, we obtain

$$\sigma_{rz} = b/r, \qquad u(r, t_1) = F(b, r), \qquad v = 0, \qquad b = c(t_1). \tag{3.1}$$

Due to the continuity of the elastic strain component on the elastoplastic boundary $r = r_1(t)$, the last two relations from (2.4) are also valid in the region of viscoplastic flow. Then, according to the Murnaghan formula (1.3), for $p_{ij} \neq 0$, the stress components in this region are given by

$$\sigma_{rr} = -(p_1 + 2\mu) - 2(b + \mu)e_{rz}^2 = -s_1(t),$$
$$\sigma_{zz} = -s_1(t) + 4\mu e_{rz}^2, \qquad \sigma_{rz} = 2\mu e_{rz}. \tag{3.2}$$

At the same time, integrating the equilibrium equations and using the continuity condition for the stress components on the elastoplastic boundary $r = r_1(t)$, we obtain the same relations for the stress components in the region of viscoplastic flow $r_0 \leqslant r \leqslant r_1(t)$ as in the elastic region.

The plastic potential (1.4) can be written as

$$f(\sigma_{rz}, \varepsilon_{rz}^p) = \sigma_{rz}^2 - (k - \eta \varepsilon_{rz}^p)^2 = 0.$$

According to the associated plastic flow law, we obtain

$$\sigma_{rz} = -k + \eta\varepsilon^p_{rz}, \qquad \lambda = \varepsilon^p_{rz}/(\eta\varepsilon^p_{rz} - k). \tag{3.3}$$

A comparison of relations (3.2) and (3.3) yields the plastic strain rate

$$\varepsilon^p_{rz} = (b/r + k)/\eta. \tag{3.4}$$

Taking into account that, on the elastoplastic boundary, $r = r_1(t)$ and $\varepsilon^p_{rz} = 0$, we have

$$r_1 = -b/k. \tag{3.5}$$

Using the kinematic relations

$$\frac{dd_{rz}}{dt} = \frac{\partial d_{rz}}{\partial t} = \frac{1}{2}\,v', \qquad r_{zr} = -r_{rz} = \frac{1}{2}\,v',$$

$$\varepsilon_{rz} = \frac{1}{2}\,v' = \varepsilon^e_{rz} + \varepsilon^p_{rz} = \frac{\partial e_{rz}}{\partial t} + \frac{\partial p_{rz}}{\partial t}, \qquad \varepsilon^p_{rr} = \frac{dp_{rr}}{dt} + 2p_{rz}(r_{zr} + \varepsilon^p_{rz}),$$

$$\varepsilon^p_{zz} = \frac{dp_{zz}}{dt} + 2p_{rz}(r_{rz} + \varepsilon^p_{rz}), \qquad \varepsilon^p_{rr} = -\varepsilon^p_{zz} = -2\varepsilon^p_{rz}e_{rz},$$

which hold in the case considered, we obtain the velocities of points in the region of viscoplastic flow:

$$v = G(b, r, r_0) + v^*, \qquad G(b, r, r_0) = 2(b\ln(r/r_0) + k(r - r_0))/\eta. \tag{3.6}$$

Using the condition of equality of the velocities (3.1) and (3.6) on the elastoplastic boundary $r = r_1(t)$, for the value of r_1 corresponding to the velocity $v^* = \alpha t_1$ on the surface $r = r_0$, we obtain the equation

$$G(b, r_1, r_0) + v^* = 0. \tag{3.7}$$

The displacement in the region of irreversible strain is found by integrating (3.6) to within an arbitrary function $q(r)$:

$$u = tG(b, r, r_0) + v^*t + q(r). \tag{3.8}$$

The function $q(r)$ should be such that the displacements in (3.1) and (3.8) and their derivatives u' at $r = r_1$ are continuous, and the displacements in (2.3) and (3.8) at $t = 0$ coincide. These conditions are satisfied by the function

$$q(r) = F(b, r). \tag{3.9}$$

Thus, the final solution of the problem of viscoplastic flow is described by relations (3.1) and (3.5) in the elastic region $r_1(t) \leqslant r \leqslant R$ and by relations (3.6), (3.8), and (3.9) in the region of viscoplastic flow. The stresses and, hence, reversible strains in the region of viscoplastic flow are calculated by the same relations as in the elastic region. Irreversible strains are found from the system of equations that (in the formulation considered) follows from formula (1.2):

$$-2e^2_{rz} + p_{rr} - 2e_{rz}p_{rz} = -0.5(u')^2, \qquad p_{zz} - 2e_{rz}p_{rz} = 0, \qquad e_{rz} + p_{rz} = 0.5u'.$$

As a result, we obtain

$$p_{rz} = kt(1 - r_1/r)/\eta, \qquad p_{rr} = 2e_{rz}(e_{rz} + p_{rz}) - (u')^2/2,$$

$$p_{zz} = 2e_{rz}p_{rz}, \qquad e_{rz} = -kr_1/(2\mu r).$$

The development of the region of viscoplastic flow $(r_1/R)(\tau)$ $(\tau = \alpha t^2/r_0)$ for $r_0/R = 0.2$, $k/\mu = 0.00621$, and $y = (\mu/\eta)\sqrt{r_0/\alpha} = 100$ is shown in Fig. 1. Figure 2 gives the distribution of the displacements u/R for $r_1/R = 0.8$.

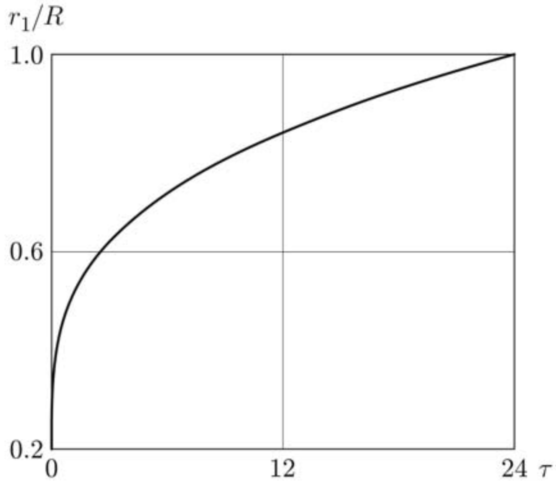

Fig. 1

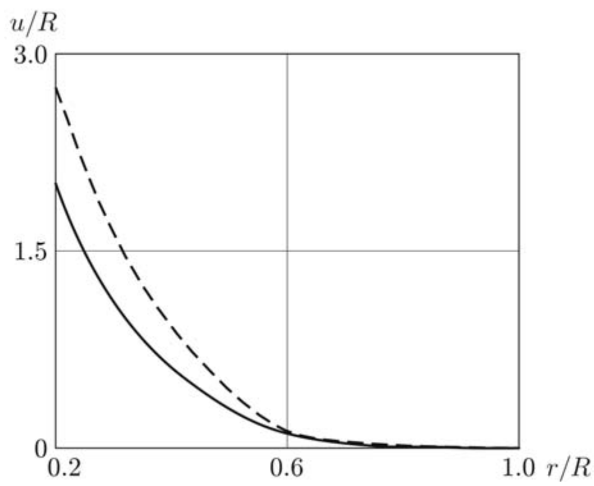

Fig. 2

Fig. 1. Law of motion of the boundary of the region of viscoplastic flow for the case of increasing velocity of motion of the rigid surface.

Fig. 2. Distributions of displacements for motion of the inner surface with velocity $v = v^*$ (solid curve) and at the deceleration time $\tau = 12.5$ (dashed curve) for $r_1/R = 0.8$ and $r_2/R = 0.6$.

4. Deceleration. Let, beginning from some time $t = t_1$, the velocity of the surface $r = r_0$ decreases, for example as

$$v = \alpha t_1 - \beta(t - t_1), \tag{4.1}$$

to zero, i.e., complete deceleration occurs at the time $t_k = (\alpha/\beta + 1)t_1$. We consider the change of the stress–strain parameters at each time $t_1 \leqslant t^* \leqslant t_k$.

Beginning from the time $t = t_1$, in the material there are three regions: a region of elastic strain $r_1 \leqslant r \leqslant R$, a region with the unchanged irreversible strain tensor $r_2(t) \leqslant r \leqslant r_1$, and a region of continuing viscoplastic flow $r_0 \leqslant r \leqslant r_2(t)$ [$r_2(t)$ is the boundary of the region]. In the first region, relations (3.1) with the current value $g = c(t^*)$ of the function $c(t)$ are valid. In the region $r_2(t) \leqslant r \leqslant r_1$ with the unchanged irreversible strain tensor, its components p_{rr} and p_{zz} should change according to the transfer equations in (1.1). In this case, the irreversible strain rates ε^p_{rr} and ε^p_{zz} ($\varepsilon^p_{rr} = -\varepsilon^p_{zz}$) also continue to change. The strain component p_{rz} ($\varepsilon^p_{rz} = 0$) remains unchanged:

$$p_{rz} = kt_1(1 - r_1/r)/\eta.$$

Determining the integration constant from the condition of equality of the displacements at $r = r_1$ and using the condition $u' = 2(e_{rz} + p_{rz})$ we find the displacement in this region:

$$u = F(g, r) + t_1 G(b, r, r_1).$$

Then, the velocity in this region is $v = \dot{u} = 0$.

In the region of continuing viscoplastic flow $r_0 \leqslant r \leqslant r_2(t)$, we obtain

$$\varepsilon^p_{rz} = (g/r + k)/\eta, \qquad r_2 = -g/k, \qquad v = G(g, r, r_0) + v^* - \beta(t^* - t_1),$$

$$u = (t - t_1)G(g, r, r_0) + v^* t - \beta t(t^* - t_1) + q_1(r).$$

From the conditions of continuity of the displacement and its derivatives for $r = r_2$, we determine $q_1(r)$ and obtain the equation for the value of r_2 corresponding to the velocity $v^* - \beta(t^* - t_1)$ of the surfaces $r = r_0$:

$$q_1(r) = t_1 G(b, r, r_1) + F(g, r), \qquad G(g, r_2, r_0) + v^* - \beta(t^* - t_1) = 0. \tag{4.2}$$

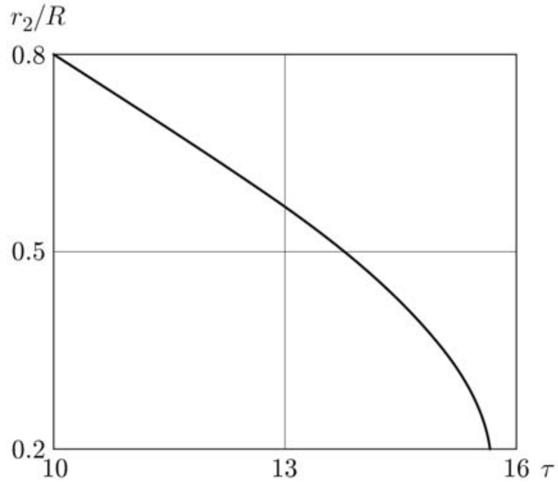

Fig. 3. Law of motion of the boundary of the region of viscoplastic flow for the case of decreasing velocity of motion of the rigid surface.

In this region, the plastic strain component p_{rz} is given by the relation

$$p_{rz} = \frac{k(t-t_1)}{\eta}\Big(1-\frac{r_2}{r}\Big) + \frac{kt_1}{\eta}\Big(1-\frac{r_1}{r}\Big).$$

Figure 3 shows the variation of the surface $r = r_2$ ($\alpha/\beta = 1/4$). It is evident that, at the final deceleration time $t = t_k$, the surface r_2 coincides with the surface $r = r_0$. From this, it follows that, in the region $r_0 \leqslant r \leqslant r_1$, the strain component p_{rz} is constant and the value of the residual stress component $\sigma_{rz} = -kr_0/r$ coincides with its value at the time of onset of plastic flow. The displacement distribution at the deceleration time ($\tau = 12.5$) is shown in Fig. 2 (dashed curve).

5. Deformation during Motion of the Outer Cylinder. We consider the deformation of an elastoviscoplastic material in the case where the outer rigid cylinder moves and the inner cylinder remains motionless:

$$u(r_0,t) = v(r_0,t) = 0, \qquad v(R,t) = \alpha t.$$

Plastic flow also begins in the vicinity of the inner rigid wall when the plasticity condition $\sigma_{rz}\Big|_{r=r_0} = k$ is satisfied. The stress components are calculated from relations (2.3), at the time of onset of plastic flow, $c = kr_0$, and a_0 is the value of the stress components σ_{rr} on the surface $r = R$. Thus, the value of the parameter u_0 of the onset of plastic flow is equal to the value of this parameter for motion of the inner surface.

With a further change in the velocity of the outer surface, the region of viscoplastic flow where $\sigma_{rz} = k+\eta\varepsilon^p_{rz}$, is given by the inequalities $r_0 \leqslant r \leqslant r_1(t)$, and in the region $r_1(t) \leqslant r \leqslant R$, the strain is reversible. In this case, for the displacements and velocities, we obtain the following relations:

$$u = (kr_1/\mu)\ln(r/r_0) + t\,G_1(b_1,r_1,r_0), \qquad v = \alpha t_1 \tag{5.1}$$

in the region $r_1(t) \leqslant r \leqslant R$;

$$u = (kr_1/\mu)\ln(r/r_0) + tG_1(b_1,r,r_0), \qquad v = G_1(b_1,r,r_0) \tag{5.2}$$

in the region $r_0 \leqslant r \leqslant r_1(t)$. In this case,

$$G_1(b_1,r_1,r_0) = 2(b_1\ln(r_1/r_0) - r_1 + r_0)/\eta, \qquad b_1 = kr_1.$$

Using the continuity condition for velocities (5.1) and (5.2) on the boundary of the region of viscoplastic flow $r = r_1$, for the motion of the inner and outer cylinders, we have identical equations of motion of the given boundary (3.7), despite different velocity and displacement components.

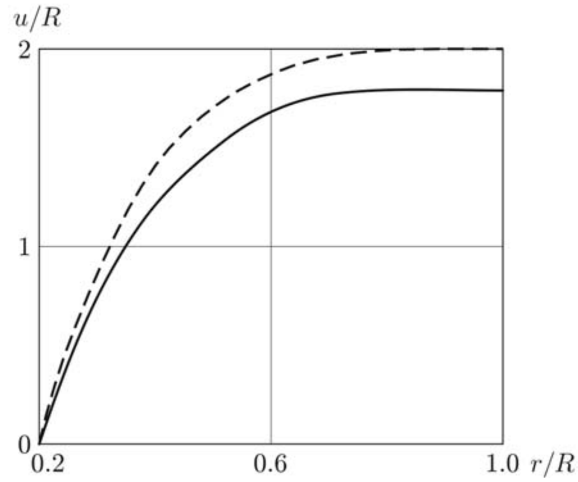

Fig. 4. Displacement distributions for motion of the outer surface with velocity $v = v^*$ (solid curve) and at the deceleration time $\tau = 10.6$ (dashed curve) for $r_1/R = 0.8$ and $r_2/R = 0.6$.

According to relation (4.1), if the velocity of motion of the outer surface changes, the displacement and velocity components are found from the following relations:

$$u = (kr_2/\mu)\ln(r/r_0) + t_1G_1(b_1, r_1, r_0) + (t - t_1)G_1(g_1, r_2, r_0), \qquad g_1 = kr_2,$$

$$v = \alpha t_1 - \beta(t^* - t_1)$$

in the region of elastic strain, $r_1 \leqslant r \leqslant R$;

$$u = (kr_2/\mu)\ln(r/r_0) + t_1G_1(b_1, r, r_0) + (t - t_1)G_1(g_1, r_2, r_0), \qquad v = \alpha t_1 - \beta(t^* - t_1) \tag{5.3}$$

in the region of unchanged irreversible strain tensor $r_2(t) \leqslant r \leqslant r_1$;

$$u = (kr_2/\mu)\ln(r/r_0) + t_1G_1(b_1, r, r_0) + (t - t_1)G_1(g_1, r, r_0), \qquad v = G_1(g_1, r, r_0) \tag{5.4}$$

in the region of viscoplastic flow $r_0 \leqslant r \leqslant r_2(t)$,

As in the case of increasing velocity of the outer cylinder, for $r = r_2$, the equation of motion of the boundary $r = r_2$ obtained from the condition of equality of velocities (5.3) and (5.4) coincides with the second equation in (4.2), despite differences in the velocity and displacement components between the cases of motion of the inner and outer cylinders.

Figure 4 shows the displacement distributions during loading and at the deceleration time.

Thus, an analytical solution was obtained for the boundary-value problem of rectilinear motion of the material in the gap between two rigid coaxial cylindrical surfaces for large elastoplastic strains.

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